Numerical Problems

Important formulas

- **Quantization Rule** $Q = ne$
- **Energy supplied by the charge** $E = q(V_a - V_b)$
- **Coulomb's Law** $F = k\frac{q_1 q_2}{r^2}$, $k = 9 \times 10^9\ Nm^2C^{-2}$
- **Electric Field Intensity** $E = \frac{F}{q_\circ}$
- **Electric Potential** $V = \frac{W}{q}$
- **Capacitor and Capacitance Formula** $Q = CV$
- **Equivalent Capacitance For Series Combination** $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
- **Equivalent Capacitance For Parallel Combination** $C_{eq} = C_1 + C_2$

13.1 The charge of how many negatively charged particles would be equal to $100\ \mu C$. Assume charge on one negative particle is $1.6 \times 10^{-19} C$? *(ALP)*

Given Data

$$Charge = Q = 100\mu C$$
$$Q = 100 \times 10^{-6} C$$
$$Charge\ on\ electron = e = 1.6 \times 10^{-19} C$$

To Find

$$No.of\ charged\ particles = n = ?$$

Solution

By using quantization rule

$$Q = ne$$
$$n = \frac{Q}{e}$$
$$n = \frac{100 \times 10^{-6} C}{1.6 \times 10^{-19} C}$$
$$\boldsymbol{n = 6.25 \times 10^{14}}$$

13.2 Two point charges $q_1 = 10\ \mu C$ and $q_2 = 5\ \mu C$ are placed at distance of $150\ cm$. What will be the Coulomb's force between them? Also find the direction of the force. *(ALP)*

Given Data

$$q_1 = 10\ \mu C$$
$$q_1 = 10 \times 10^{-6} C$$
$$q_2 = 5\ \mu C$$
$$q_2 = 5 \times 10^{-6} C$$
$$r = 150\ cm$$
$$r = 150 \times 10^{-2} m$$
$$r = 1.5\ m$$

To Find

$$Coulomb's\ force = F = ?$$
$$Direction\ of\ force = ?$$

Solution

By using formula of coulomb's law

$$F = k\frac{q_1 q_2}{r^2}$$
$$F = (9 \times 10^9)\frac{(10 \times 10^{-6})(5 \times 10^{-6})}{(1.5)^2}$$
$$F = \frac{0.45}{2.25}$$
$$\boldsymbol{F = 0.2\ N}$$

Direction: As both charges are same so force between two charges is repulsive (force of repulsion)

13.3 The force of repulsion between two identical positive charges is $0.8\ N$, when the charges are $0.1\ m$ apart. Find the value of each charge. *(ALP)*

Given Data

As both charges are identical, so

$$q_1 = q_2 = q$$
$$Force = F = 0.8\ N$$
$$Distance = r = 0.1\ m$$

To Find

$$Value\ of\ each\ charge = q = ?$$

Solution

By using formula of coulomb's law

$$F = k\frac{q_1 q_2}{r^2}$$
$$F = k\frac{(q)(q)}{r^2}$$
$$F = \frac{kq^2}{r^2}$$
$$q^2 = \frac{Fr^2}{k}$$
$$q^2 = \frac{(0.8)(0.1)^2}{9 \times 10^9}$$
$$q^2 = 8.89 \times 10^{-13}$$
$$\sqrt{q^2} = \sqrt{8.89 \times 10^{-13}}$$
$$\boldsymbol{q = 9.42 \times 10^{-7}\ C}$$

13.4 Two charges repel each other with a force of $0.1\ N$ when they are $5\ cm$ apart. Find the forces between the same charges when they are $2\ cm$ apart. *(ALP)*

Given Data

As both charges repel each other, they are similar charges, so

$$q_1 = q_2 = q$$
$$F_1 = 0.1\ N$$
$$r_1 = 5\ cm$$
$$r_1 = \frac{5}{100} m$$
$$r_1 = 0.05m$$

To Find

$$F_2 = ? \quad when\ r_2 = 2\ cm = 0.02m$$

Solution

By using formula of coulomb's law

$$F_1 = k\frac{q_1 q_2}{{r_1}^2}$$
$$F_1 = k\frac{(q)(q)}{{r_1}^2}$$
$$F_1 = \frac{kq^2}{{r_1}^2}$$
$$q^2 = \frac{F_1 {r_1}^2}{k}$$
$$q^2 = \frac{(0.1)(0.05)^2}{9 \times 10^9}$$
$$q^2 = 2.78 \times 10^{-14}$$
$$\sqrt{q^2} = \sqrt{2.78 \times 10^{-14}}$$
$$\boldsymbol{q = 1.67 \times 10^{-7}\ C}$$

Now for F_2

$$F_2 = k\frac{q_1 q_2}{{r_2}^2}$$

$$F_2 = k\frac{(q)(q)}{{r_2}^2}$$
$$F_2 = \frac{kq^2}{{r_2}^2}$$
$$F_2 = \frac{(9\times10^9)(1.67\times10^{-7})^2}{(0.02)^2}$$
$$F_2 = 6.22\times10^{-1}\ N$$
$$\boldsymbol{F_2 = 0.62\ N}$$

13.5 The electric potential at a point in an electric field in$10^4\ V$. If a charge of $+100\ \mu C$ is brought from infinity to this point. What would be the amount of work done on it? *(ALP)*

Given Data

$$Electric\ potential = V = 10^4\ V$$
$$Charge = q = 100\ \mu C$$
$$q = 100\times10^{-6}\ C$$

To Find

$$Work\ done = W = ?$$

Solution

By using formula of electric potential

$$V = \frac{W}{q}$$
$$W = qV$$
$$W = (100\times10^{-6})(10^4)$$
$$\boldsymbol{W = 1\ J}$$

13.6 A point charge of $+2\ C$ is transferred from a point at potential $100\ V$ to a point at potential $50\ V$, what would be the energy supplied by the charge? *(ALP)*

Given Data

$$Charge = Q = 2\ C$$
$$Higher\ ptential = V_a = 100\ V$$
$$Lower\ ptential = V_b = 50\ V$$

To Find

$$Ebergy\ supplied\ by\ charge = E = ?$$

Solution

By using formula of energy supplied

$$E = q(V_a - V_b)$$
$$E = 2(100-50)$$
$$E = 2(50)$$
$$\boldsymbol{E = 100\ J}$$

13.7 A capacitor holds $0.06\ coulombs$ of charge when fully charged by a $9\ volt$ battery. Calculate capacitance of the capacitor. *(ALP)*

Given Data

$$Charge\ = Q = 0.06\ C$$
$$Voltage = V = 9\ V$$

To Find

$$Capacitance\ of\ capacitor = C = ?$$

Solution

By using formula of capacitance

$$Q = CV$$
$$C = \frac{Q}{V}$$
$$C = \frac{0.06}{9}$$
$$\boldsymbol{C = 6.67\times10^{-3}\ F}$$

13.8 A capacitor holds $0.03\ coulombs$ of charge when fully charged by a $6\ volt$ battery. How much voltage would be required for it to hold $2\ coulombs$ of charge? *(ALP)*

Given Data

$$Q_1 = 0.03\ C$$
$$V_1 = 6\ V$$
$$Q_2 = 2\ C$$

To Find

$$V_2 = ?$$

Solution

By using formula of capacitance

$$Q = CV$$
$$C = \frac{Q}{V}$$

Capacitance for 1st capacitor

$$C = \frac{Q_1}{V_1} \quad \ldots(i)$$

Capacitance for 2nd capacitor

$$C = \frac{Q_2}{V_2} \ldots(ii)$$

Comparing equation (i) and (ii)

$$\frac{Q_1}{V_1} = \frac{Q_2}{V_2}$$
$$Q_1V_2 = Q_2V_1$$
$$V_2 = \frac{Q_2V_1}{Q_1}$$
$$V_2 = \frac{(2)(6)}{0.03}$$
$$\boldsymbol{V_2 = 400\ V}$$

13.9. Two capacitors of capacitance $6\mu F$ and $12\mu F$ are connected in series with a $12V$ battery. Find the equivalent capacitance of the combination. Find the charge and their potential difference across each capacitor. *(ALP)*

Given Data

$$C_1 = 6\ \mu F$$
$$C_2 = 12\ \mu F$$
$$V = 12\ V$$

To Find

$$C_{eq} = ?$$
$$Q_1 = ?, \quad Q_2 = ?$$
$$V_1 = ?, \quad V_2 = ?$$

Solution

By using formula of equivalent capacitance for series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$\frac{1}{C_{eq}} = \frac{1}{6\mu F} + \frac{1}{12\mu F}$$
$$\frac{1}{C_{eq}} = \frac{2+1}{12\mu F}$$
$$\frac{1}{C_{eq}} = \frac{3}{12\mu F}$$
$$\frac{1}{C_{eq}} = \frac{1}{4\mu F}$$
$$\boldsymbol{C_{eq} = 4\ \mu F}$$

Charge will be same for each capacitor in series combination, so $Q = Q_1 = Q_2$

$$Q = C_{eq}V$$
$$= (4\ \mu F)(12\ V)$$
$$\boldsymbol{Q = 48\ \mu C}$$

Voltage across 1st capacitor

$$Q = C_1V_1$$
$$V_1 = \frac{Q}{C_1}$$
$$V_1 = \frac{48\ \mu C}{6\ \mu F}$$
$$\boldsymbol{V_1 = 8\ V}$$

Voltage across 2nd capacitor

$$Q = C_2V_2$$
$$V_2 = \frac{Q}{C_2}$$
$$V_2 = \frac{48\ \mu C}{12\ \mu F}$$
$$\boldsymbol{V_2 = 4\ V}$$

13.10. Two capacitors of capacitances of $6\mu F$ and $12\mu F$ are connected in parallel with a $12V$ battery. Find the equivalent capacitance of the combination. Find the charge and the potential difference across each capacitor. *(ALP)*

Given Data

$$C_1 = 6\ \mu F$$
$$C_2 = 12\ \mu F$$
$$V = 12\ V$$

To Find

$$C_{eq} = ?$$
$$Q_1 = ?, \quad Q_2 = ?$$
$$V_1 = ?, \quad V_2 = ?$$

Solution

By using formula of equivalent capacitance for parallel combination

$$C_{eq} = C_1 + C_2$$
$$C_{eq} = 6\ \mu F + 12\ \mu F$$
$$\boldsymbol{C_{eq} = 18\ \mu F}$$

As potential difference (voltage) will be same for each capacitor in parallel combination, so

$$\boldsymbol{V = V_1 = V_2 = 12\ V}$$

Charge across 1st capacitor

$$Q_1 = C_1V_1$$
$$Q_1 = (6\ \mu F)(12\ V)$$
$$\boldsymbol{Q_1 = 72\ \mu C}$$

Charge across 2nd capacitor

$$Q_2 = C_2V_2$$
$$Q_2 = (12\ \mu F)(12\ V)$$
$$\boldsymbol{Q_2 = 144\ \mu C}$$

Examples

13.1 Two bodies are oppositely charged with $500\ \mu C$ and $100\ \mu C$ charge. Find the force between the two charges if the distance between them in air is $0.5\ m$. *(ALP)*

Given Data

$$q_1 = 500\ \mu C$$
$$q_1 = 500 \times 10^{-6}\ C$$
$$q_2 = 100\ \mu C$$
$$q_2 = 100 \times 10^{-6}\ C$$
$$r = 0.5\ m$$

To Find

$$Force = F = ?$$

Solution

By using formula of coulomb's law

$$F = k\frac{q_1q_2}{r^2}$$
$$F = (9 \times 10^9)\frac{(500 \times 10^{-6})(100 \times 10^{-6})}{(0.5)^2}$$
$$F = \frac{450}{0.25}$$
$$\boldsymbol{F = 1800\ N}$$

13.2 The capacitance of a parallel plate capacitor is $100\ \mu F$. If the potential difference between its plates is $50\ volts$, find the quantity of charge stored on each plate. *(ALP)*

Given Data

$$Capacitance = C = 100\ \mu F$$
$$C = 100 \times 10^{-6}\ F$$
$$Voltage = V = 50\ V$$

To Find

$$Charge = Q = ?$$

Solution

By using formula of capacitance

$$Q = CV$$
$$Q = (100 \times 10^{-6})(50)$$
$$Q = 5 \times 10^{-3}\ C$$
$$\boldsymbol{Q = 5\ mC}$$

13.3 Three capacitors with capacitances of $3.0\ \mu F$, $4.0\ \mu F$, and $5.0\ \mu F$ are arranged in parallel combination with a battery of $6\ V$, where $1\ \mu F = 10^{-6}\ F$. Find (a) the total capacitance (b) the voltage across each capacitor (c) the quantity of charge on each plate of the capacitor. *(ALP)*

Given Data

$$C_1 = 3.0\ \mu F$$
$$C_2 = 4.0\ \mu F$$
$$C_3 = 5.0\ \mu F$$
$$V = 6\ V$$

To Find

$$C_{eq} = ?$$
$$V_1 = ?, V_2 = ?, V_3 = ?$$
$$Q_1 = ?, Q_2 = ?, Q_3 = ?$$

Solution

By using formula of equivalent capacitance for parallel combination

$$C_{eq} = C_1 + C_2 + C_3$$
$$C_{eq} = 3.0\ \mu F + 4.0\ \mu F + 5.0\ \mu F$$
$$\boldsymbol{C_{eq} = 12\ \mu F}$$

As potential difference (voltage) will be same for each capacitor in parallel combination, so

$$\boldsymbol{V = V_1 = V_2 = V_3 = 6\ V}$$

Charge across 1st capacitor

$$Q_1 = C_1V_1$$
$$Q_1 = (3.0\ \mu F)(6\ V)$$
$$\boldsymbol{Q_1 = 18\ \mu C}$$

Charge across 2nd capacitor

$$Q_2 = C_2V_2$$
$$Q_2 = (4.0\ \mu F)(6\ V)$$
$$\boldsymbol{Q_2 = 24\ \mu C}$$

Charge across 3rd capacitor

$$Q_2 = C_2V_2$$
$$Q_2 = (5.0\ \mu F)(6\ V)$$
$$\boldsymbol{Q_2 = 30\ \mu C}$$

13.4 Three capacitors with capacitances of $3.0\ \mu F$, $4.0\ \mu F$, and $5.0\ \mu F$ are arranged in series combination to a battery of $6\ V$, where $1\ \mu F = 10^{-6}\ F$. Find (a) the total capacitance of the series combination. (b) the quantity of charge across each capacitor. (c) the voltage across each capacitor. ***(ALP)***

Given Data

$$C_1 = 3.0\ \mu F$$
$$C_2 = 4.0\ \mu F$$
$$C_3 = 5.0\ \mu F$$
$$V = 6\ V$$

To Find

$$C_{eq} = ?$$
$$V_1 = ?, V_2 = ?, V_3 = ?$$
$$Q_1 = ?, Q_2 = ?, Q_3 = ?$$

Solution

By using formula of equivalent capacitance for series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$\frac{1}{C_{eq}} = \frac{1}{3.0\ \mu F} + \frac{1}{4.0\ \mu F} + \frac{1}{5.0\ \mu F}$$
$$\frac{1}{C_{eq}} = \frac{20 + 15 + 12}{60\ \mu F}$$
$$\frac{1}{C_{eq}} = \frac{47}{60\ \mu F}$$
$$C_{eq} = \frac{60\ \mu F}{47}$$
$$\boldsymbol{C_{eq} = 1.3\ \mu F}$$

Charge will be same for each capacitor in series combination, so $Q = Q_1 = Q_2 = Q_3$

$$Q = C_{eq}V$$
$$Q = (1.3\ \mu F)(6\ V)$$
$$\boldsymbol{Q = 7.8\ \mu C}$$

Voltage across 1st capacitor

$$Q = C_1V_1$$
$$V_1 = \frac{Q}{C_1}$$
$$V_1 = \frac{7.8\ \mu C}{3.0\ \mu F}$$
$$\boldsymbol{V_1 = 2.6\ V}$$

Voltage across 2nd capacitor

$$Q = C_2V_2$$
$$V_2 = \frac{Q}{C_2}$$
$$V_2 = \frac{7.8\ \mu C}{4.0\ \mu F}$$
$$\boldsymbol{V_2 = 1.95\ V}$$

Voltage across 3rd capacitor

$$Q = C_3V_3$$
$$V_3 = \frac{Q}{C_3}$$
$$V_3 = \frac{7.8\ \mu C}{5.0\ \mu F}$$
$$\boldsymbol{V_3 = 1.56\ V}$$