#### Unit 12

## **Geometrical Optics**

## **Numerical Problems**

## Important formulas:

- Mirror Formula  $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$
- Relationship Between Radius of Curvature and Focal Length  $f = \frac{R}{2}$
- > Formula of Magnification

$$M = \frac{h_i}{h_\circ} = \frac{q}{p} = \frac{Image\ height}{Object\ height}$$

Refractive Index

$$Refrective\ Index = \frac{\textit{Speed of light in air}}{\textit{Speed of light in medium}}$$

- R.I. of 1<sup>st</sup> medium (air) w.r.t 2<sup>nd</sup> medium (glass) (Snell's Law)  $n=\frac{\sin \angle i}{\sin \angle r}$
- $\triangleright$  R.I. of glass w.r.t air (Snell's Law)

$$n = \frac{\sin \angle r}{\sin \angle i}$$

- > Total Refractive Index  $n = \frac{n_1}{n_2}$
- ightharpoonup Critical Angle  $\sin \angle c = \frac{1}{n}$
- Power of Lens

$$Power of lens = \frac{1}{focal lenght in metres}$$

 $> 1D = 1m^{-1}$ 

## For Mirrors:

- $\triangleright$  Always (+ve f) for **concave** (converging) mirror
- Always  $(-ve\ f)$  for **convex** (diverging) mirror
- $\triangleright$  Real image always lie in front of mirror (+q)
- $\triangleright$  Virtual image always lie behind the mirror  $(\neg q)$

#### For Lenses:

- Always (+ve f) for convex (converging) lens
- $\triangleright$  Always  $(-ve\ f)$  for **concave** (diverging) lens
- Real image always lie on other side of object (Right side of lens) (+q)
- $\succ$  Virtual image always lie on same side of object (left side of lens) (-q)

Note: When the image distance is positive, the image is on the same side of the mirror as the object, and it is real and inverted. When the image distance is negative, the image is behind the mirror, so the image is virtual and upright. A negative M means that the image is inverted. Positive M means an upright image.

12.1. An object  $10.0 \ cm$  in front of a convex mirror forms an image  $5.0 \ cm$  behind the mirror. What is the focal length of the mirror? (ALP)

#### **Given Data**

Object distance = 
$$p = 10 cm$$
  
Image distance =  $q = -5 cm$ 

: image behind the mirror

To Find

Focal lenght = 
$$f = ?$$

### Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{-5}$$

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{5}$$

$$\frac{1}{f} = \frac{1-2}{10}$$

$$\frac{1}{f} = \frac{-1}{10}$$

$$f = -10 cm$$

12.2. An object  $30\ cm$  tall is located  $10.5\ cm$  from a concave mirror with focal length  $16\ cm$ . (a) where is the image located? (b) How high is it? *(ALP)* Given Data

$$\begin{array}{l} \textit{Object height} = h_o = 30 \, \textit{cm} \\ \textit{Object distance} = p = 10.5 \, \textit{cm} \\ \textit{Focal Lenght} = f = 16 \, \textit{cm} \\ \end{array}$$

To Find

Image distance = 
$$q = ?$$
  
Image height =  $h_i = ?$ 

#### Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{16} - \frac{1}{10.5}$$

$$\frac{1}{q} = \frac{10.5 - 16}{(16)(10.5)}$$

$$\frac{1}{q} = \frac{-5.5}{168}$$

$$q = -\frac{168}{5.5}$$

$$q = -30.54 cm$$

Negative sign shows that the **image is virtual** and located **behind the mirror**.

Now by using formula of magnification

$$M = \frac{h_i}{h_0} = \frac{q}{p}$$

$$\Rightarrow \frac{h_i}{h_0} = \frac{q}{p}$$

$$h_i = \frac{q}{p} \times h_0$$

$$h_i = \frac{30.54}{10.5} \times 30$$

$$h_i = 87.3 cm$$

12.3. An object and its image in a concave mirror are of the same height, yet inverted, when the object is  $20\ cm$  from the mirror. What is the focal length of the mirror? (ALP)

**Given Data** 

$$h_{\circ} = h_{i}$$
Object distance =  $p = 20 \ cm$ 

To Find

$$Focal\ length = f = ?$$

Solution

By using formula of magnification

$$M = \frac{h_i}{h_{\circ}} = \frac{q}{p}$$

$$\Rightarrow \frac{h_i}{h_{\circ}} = \frac{q}{p}$$

$$\frac{h_i}{h_i} = \frac{q}{p} \quad \because h_{\circ} = h_i$$

$$1 = \frac{q}{p}$$

$$p = q$$

Here, q will be taken as +ive because mirror is concave. Now by using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p}$$

$$\frac{1}{f} = \frac{1+1}{p}$$

$$\frac{1}{f} = \frac{2}{p}$$

$$f = \frac{p}{2}$$

$$f = \frac{10}{2} cm$$

12.4. Find the focal length of a mirror that forms an image  $5.66\ cm$  behind the mirror of an object placed at  $34.4\ cm$  in front of the mirror. Is the mirror concave or convex? *(ALP)* 

#### **Given Data**

Object distance = 
$$p = 34.4 \text{ cm}$$
  
Image distance =  $q = -5.66 \text{ cm}$   
 $\because$  image behind the mirror

### To Find

$$Focal\ length = f = ?$$

## Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{34.4} + \frac{1}{-5.66}$$

$$\frac{1}{f} = \frac{1}{34.4} - \frac{1}{5.66}$$

$$\frac{1}{f} = \frac{5.66 - 34.4}{(34.4)(5.66)}$$

$$\frac{1}{f} = \frac{-28.74}{194.704}$$

$$f = -\frac{194.704}{28.74}$$

$$f = -6.77 cm$$

Negative sign of focal length shows that it is a **convex mirror**.

12.5. An image of a statue appears to be  $11.5\ cm$  behind a concave mirror with focal length  $13.5\ cm$ . Find the distance from the statue to the mirror. (ALP)

**Given Data** 

Image distance = 
$$q = -11.5$$
 cm  
 $\because$  image behind the mirror  
Focal lenght =  $f = 13.5$  cm

To Find

Ditance of object = 
$$p = ?$$

#### Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{13.5} - \frac{1}{-11.5}$$

$$\frac{1}{p} = \frac{1}{13.5} + \frac{1}{11.5}$$

$$\frac{1}{p} = \frac{11.5 + 13.5}{(13.5)(11.5)}$$

$$\frac{1}{p} = \frac{25}{155.25}$$

$$p = \frac{25}{25}$$

$$p = 6.21 cm$$

12.5. An image of a statue appears to be  $11.5\ cm$  behind a convex mirror with focal length  $13.5\ cm$ . Find the distance from the statue to the mirror. (ALP)

# **Given Data**

Image distance = 
$$q = -11.5$$
 cm  
 $\because$  image behind the mirror  
Focal lenght =  $f = -13.5$  cm  
 $\because$  for convex mirror (-ive f)

To Find

Distance of object = 
$$p = ?$$

#### Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{-13.5} - \frac{1}{-11.5}$$

$$\frac{1}{p} = -\frac{1}{13.5} + \frac{1}{11.5}$$

$$\frac{1}{p} = \frac{-11.5 + 13.5}{(13.5)(11.5)}$$

$$\frac{1}{p} = \frac{2}{155.25}$$

$$p = \frac{155.25}{2}$$

$$p = 77.62 cm$$

12.6. An image is produced by a concave mirror of focal length  $8.7\ cm$ . The object is  $13.2\ cm$  tall and at a distance  $19.3\ cm$  from the mirror. (a) Find the location and height of the image. (b) Find the height of the image produced by the mirror if the object is twice as far from the mirror.

**Given Data** 

$$Focal\ length = f = 8.7\ cm$$

Object height = 
$$h_0$$
 = 13.2 cm  
Object distance =  $p$  = 19.3 cm

#### To Find

Image distance = 
$$q = ?$$
  
Image height =  $h_i = ?$   
Image height =  $h_i = ?$  When  $p = 2p$ 

#### Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{8.7} - \frac{1}{19.3}$$

$$\frac{1}{q} = \frac{19.3 - 8.7}{(8.7)(19.3)}$$

$$\frac{1}{q} = \frac{10.6}{167.91}$$

$$q = \frac{167.91}{10.6}$$

$$q = 15.84 cm$$

Now

$$M = \frac{h_i}{h_0} = \frac{q}{p}$$

$$\Rightarrow \frac{h_i}{h_0} = \frac{q}{p}$$

$$h_i = \frac{q}{p} \times h_0$$

$$h_i = \frac{15.84}{19.3} \times 13.2$$

$$h_i = 10.83 cm$$

When p = 2p

$$h_i = \frac{q}{2p} \times h_0$$
 $h_i = \frac{15.84}{2(19.3)} \times 13.2$ 
 $h_i = 5.42 \ cm$ 

12.7. Nabeela uses a concave mirror when applying makeup. The mirror has a radius of curvature of 38 cm. (a) What is the focal length of the mirror? (b) Nabeela is located 50 cm from the mirror. Where will her image appear? (c) Will the image be upright or inverted?

Given Data

Radius of curvature = R = 38 cmDistance from mirror = p = 50 cm

To Find

Focal length = 
$$f = ?$$
  
Image distance =  $q = ?$   
Image is upright or inverted =  $?$ 

Solution

By using formula

$$f = \frac{R}{2}$$

$$f = \frac{38}{2}$$

$$f = 19 \text{ cm}$$

Now by using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{19} - \frac{1}{50}$$

$$\frac{1}{q} = \frac{50 - 19}{(19)(50)}$$

$$\frac{1}{q} = \frac{31}{950}$$

$$q = \frac{950}{31}$$

$$q = 30.64 cm (+ive)$$

When the image distance is positive, the image is real and inverted.

12.8. An object  $4\ cm$  high is placed at a distance of  $12\ cm$  from a convex lens of focal length  $8\ cm$ . Calculate the position and size of the image. Also state the nature of the image. (ALP) Given Data

Object height = 
$$h_0 = 4$$
 cm  
object distance =  $p = 12$  cm  
Focal length =  $f = 8$  cm

To Find

Image distance = 
$$q = ?$$
  
Image height =  $h_i = ?$ 

# Solution

By using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{8} - \frac{1}{12}$$

$$\frac{1}{q} = \frac{3 - 2}{24}$$

$$\frac{1}{q} = \frac{1}{24}$$

$$q = 24 cm$$

Now

$$M = \frac{h_i}{h_0} = \frac{q}{p}$$

$$\Rightarrow \frac{h_i}{h_0} = \frac{q}{p}$$

$$h_i = \frac{q}{p} \times h_0$$

$$h_i = \frac{24}{12} \times 4$$

$$h_i = \mathbf{8} \ \mathbf{cm}$$

The image obtained in this **convex lens is real, inverted** and its height shows that it is **magnified**.

12.9. An object  $10\ cm$  high is placed at a distance of  $20\ cm$  from a concave lens of focal length  $15\ cm$ . Calculate the position and size of the image. Also, state the nature of the image. (ALP)

**Given Data** 

Object height = 
$$h_0$$
 = 10 cm  
object distance =  $p$  = 20 cm

Focal length = 
$$f = -15 cm$$
  
: for concave lens (-ive f)

To Find

Image distance = 
$$q = ?$$
  
Image height =  $h_i = ?$ 

#### Solution

By using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{-15} - \frac{1}{20}$$

$$\frac{1}{q} = -\frac{1}{15} - \frac{1}{20}$$

$$\frac{1}{q} = \frac{-4 - 3}{60}$$

$$\frac{1}{q} = \frac{-7}{60}$$

$$q = -\frac{60}{7}$$

$$q = -8.57 cm$$

Now

$$M = \frac{h_i}{h_0} = \frac{q}{p}$$

$$\Rightarrow \frac{h_i}{h_0} = \frac{q}{p}$$

$$h_i = \frac{q}{p} \times h_0$$

$$h_i = \frac{8.57}{20} \times 10$$

$$h_i = 4.28 cm$$

The image obtained in this **concave lens is virtual, erect** and its height shows that it is **diminished.** 

12.10. A convex lens of focal length 6 cm is to be used to form a virtual image three times the size of the object. Where must the lens be placed? (ALP) Given Data

Focal length = 
$$f = 6 cm$$
  
Magnification =  $M = 3$ 

To Find

Lens distance = 
$$p = ?$$

## Solution

By using magnification formula

$$M = \frac{q}{p}$$
$$3 = \frac{q}{p}$$
$$q = 3p$$

Here  $m{q}$  will be taken as **negative** as the **image is virtual** i.e.  $m{q} = -3 \, m{p}$ 

Now by using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{6} = \frac{1}{p} + \frac{1}{-3p}$$

$$\frac{1}{6} = \frac{1}{p} - \frac{1}{3p}$$

$$\frac{1}{6} = \frac{3-1}{3p}$$

$$\frac{1}{6} = \frac{2}{3p}$$

$$3p = 12$$

$$p = \frac{12}{3}$$

$$p = 4 cm$$

12.11. A ray of light from air is incident on a liquid surface at an angle of incidence  $35^{\circ}$ . Calculate the angle of refraction if the refractive index of the liquid is 1.25. Also calculate the critical angle between the liquid air inter-face.

**Given Data** 

Angle of incidence = 
$$\angle i = 35^{\circ}$$
  
Refractive index of liquid =  $n = 1.25$ 

To Find

Angle of refraction = 
$$\angle r = ?$$
  
Critical angle =  $\angle C = ?$ 

#### Solution

By using Snell's law

$$n = \frac{\sin \angle t}{\sin \angle r}$$

$$\sin \angle r = \frac{\sin \angle t}{n}$$

$$\sin \angle r = \frac{\sin 35}{1.25}$$

$$\sin \angle r = \frac{0.5735}{1.25}$$

$$\sin \angle r = 0.4588$$

$$\angle r = \sin^{-1}(0.4588)$$

$$\angle r = 27.3^{\circ}$$

When light enters in air from water, Snell's law becomes

$$n = \frac{\sin \angle r}{\sin \angle i}$$
For critical angle  $\angle r = 90^{\circ}$  and  $\angle i = \angle c$ , so
$$n = \frac{\sin 90^{\circ}}{\sin \angle c}$$

$$n = \frac{1}{\sin \angle c}$$

$$\sin \angle c = \frac{1}{n}$$

$$\sin \angle c = \frac{1}{1.25}$$

$$\sin \angle c = 0.8$$

$$\angle c = \sin^{-1}(0.8)$$

$$\angle c = 53.13^{\circ}$$

## Alternate method for critical angle

$$\sin \angle c = \frac{1}{n}$$

$$\sin \angle c = \frac{1}{1.25}$$

$$\sin \angle c = 0.8$$

$$\angle c = \sin^{-1}(0.8)$$

$$\angle c = 53.13^{\circ}$$

12.12. The power of a convex lens is  $5\,D$ . At what distance the object should be placed from the lens so that its real and  $2\,times$  larger image is formed. Given Data

Power of lens = P = 5 DMagnification = M = 2

To Find

*Object distance* = p = ?

#### Solution

By using formula of power

$$P = \frac{1}{f (in meters)}$$

$$5D = \frac{1}{f}$$

$$f = \frac{1}{5D}$$

$$f = \frac{1}{5m^{-1}} \quad \because 1D = 1m^{-1}$$

$$f = \frac{1}{5}m$$

$$f = 0.2 m$$

$$f = 0.2 \times 100 cm$$

$$f = 20 cm$$

For **Convex lens** we will take f is +iveNow by using magnification formula

$$M = \frac{q}{p}$$
$$2 = \frac{q}{p}$$
$$q = 2p$$

Here q will be taken as +ive as the image is real i.e.

$$q=2p$$

Now by using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{20} = \frac{1}{p} + \frac{1}{2p}$$

$$\frac{1}{20} = \frac{2+1}{2p}$$

$$\frac{1}{20} = \frac{3}{2p}$$

$$2p = 60$$

$$p = \frac{60}{2}$$

$$p = 30 cm$$

2	10, 6
3	5, 3
5	5, 1
Sv	1, 1
2×	3×5=30

# **Examples**

12.1 A convex mirror is used to reflect light from an object placed  $66 \ cm$  in front of the mirror. The focal length of the mirror is  $46 \ cm$ . Find the location of the image. *(ALP)* 

**Given Data** 

Object distance = 
$$p = 66 \text{ cm}$$
  
Focal Lenght =  $f = -46 \text{ cm}$   
 $\because$  for convex mirror  $(-ive\ f)$ 

To Find

 $Image\ distance = q = ?$ 

Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{-46} - \frac{1}{66}$$

$$\frac{1}{q} = -\frac{1}{46} - \frac{1}{66}$$

$$\frac{1}{q} = \frac{-33 - 23}{1518}$$

$$\frac{1}{q} = \frac{-56}{1518}$$

$$q = -\frac{1518}{56}$$

$$q = -27.11 cm$$

 $2 \times 23 \times 33 = 1518$ 

Negative sign shows that the **image is virtual** and located **behind the mirror**.

12.2 An object is placed  $6 \ cm$  in front of a concave mirror that has focal length  $10 \ cm$ . Determine the location of the image. (ALP)

**Given Data** 

Object distance = 
$$p = 6$$
 cm  
Focal Lenght =  $f = 10$  cm

To Find

 $Image\ distance = q = ?$ 

Solution

By using mirror formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10} - \frac{1}{6}$$

$$\frac{1}{q} = \frac{3 - 5}{30}$$

$$\frac{1}{q} = \frac{-2}{30}$$

$$q = -\frac{30}{2}$$

$$q = -15 cm$$

Negative sign shows that the **image is virtual** and located **behind the mirror**.

12.3 A ray of light enters from air into glass. The angle of incidence is  $30^{\circ}$ . If the refractive index of glass is 1.52, then find the angle of refraction 'r'. (ALP)

**Given Data** 

Angle of incidence = 
$$\angle i = 30^{\circ}$$
  
Refractive index =  $n = 1.52$ 

To Find

*Angle of refraction* = 
$$\angle r$$
 = ?

Solution

By using Snell's law

$$n = \frac{\sin \angle i}{\sin \angle r}$$
$$\sin \angle r = \frac{\sin \angle i}{n}$$

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$$\sin \angle r = \frac{\sin 30^{\circ}}{1.52}$$

$$\sin \angle r = \frac{0.5}{1.52}$$

$$\sin \angle r = 0.33$$

$$\angle r = \sin^{-1}(0.33)$$

$$\angle r = 19.3^{\circ}$$

12.4 Find the value of critical angle for water (refracted angle =  $90^{\circ}$ ). The refractive index of water is 1.33 and that of air is 1. (ALP) **Given Data** 

Refracted angle = 
$$\angle r = 90^{\circ}$$
  
Refractive index of water =  $n = 1.33$ 

To Find

Critical angle = 
$$\angle c = ?$$

### Solution

When light enters in air from water, Snell's law becomes

$$n = \frac{\sin 2t}{\sin \angle t}$$
For critical angle  $\angle r = 90^{\circ}$  and  $\angle t = \angle c$ , so
$$n = \frac{\sin 90^{\circ}}{\sin \angle c}$$

$$n = \frac{1}{\sin \angle c}$$

$$\sin \angle c = \frac{1}{n}$$

$$\sin \angle c = \frac{1}{1.33}$$

$$\sin \angle c = 0.752$$

$$\angle c = \sin^{-1}(0.752)$$

$$\angle c = 48.8^{\circ}$$

12.5 A person 1.7 m tall is standing 2.5 m in front of a camera. The camera uses a convex lens whose focal length is  $0.05 \, m$ . Find the image distance (the distance between the lens and the film) and determine whether the image is real or virtual. (ALP) **Given Data** 

Object distance = 
$$p = 2.5 m$$
  
Focal length =  $f = 0.05 m$ 

To Find

$$Image\ distance = q = ?$$

#### Solution

By using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{0.05} - \frac{1}{1.7}$$

$$\frac{1}{q} = \frac{2.5 - 0.05}{(0.05)(2.5)}$$

$$\frac{1}{q} = \frac{2.45}{0.125}$$

$$q = \frac{0.125}{2.45}$$

$$q = 0.05 m$$

Since the image distance is positive, so a real image is formed on the film at the focal point of the lens.

12.6 A concave lens has focal length of 15 cm. At what distance should the object from the lens be placed so that it forms an image at  $10\ cm$  from the lens? Also find the magnification of the lens. (ALP) **Given Data** 

Focal length = 
$$f = -15 \text{ cm}$$
  
 $\because \text{ for concave lens } (-\text{ive } f)$ 

*Image distance* = q = -10 *cm* 

: Concave lens always forms virtual image

To Find

Object distance = 
$$p = ?$$
  
Magnification =  $M = ?$ 

#### Solution

By using lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$\frac{1}{p} = -15 - \frac{1}{-10}$$

$$\frac{1}{p} = -\frac{1}{15} + \frac{1}{10}$$

$$\frac{1}{p} = \frac{-2+3}{30}$$

$$\frac{1}{p} = \frac{1}{30}$$

$$p = \frac{30}{1}$$

$$p = 30 cm$$

Now

$$M = \frac{h_i}{h_0} = \frac{q}{p}$$

$$\Rightarrow M = \frac{q}{p}$$

$$M = \frac{10}{30}$$

$$M = \frac{1}{3}$$

The image is reduced to one-third in size than the object.

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