Unit 10 Simple Harmonic Motion and Waves

Numerical Problems

Important formulas:

- **Time period of simple pendulum** $T = 2\pi\sqrt{\frac{l}{g}}$
- **Speed of wave** $v = f\lambda$
- **Time period** $T = \frac{1}{f}$
- **Speed** $v = \frac{d}{t}$

10.1 The time period of a simple pendulum is $2\ s$. What will be its length on the Earth? What will be its length on the Moon if $g_m = {}^{g_e}/_{6}$? where $= 10\ ms^{-2}$. *(ALP)*

Given Data

$$Time\ period = T = 2\ s$$
$$Value\ of\ g\ on\ Earth = g_e = 10\ ms^{-2}$$
$$Value\ of\ g\ on\ moon = g_m = \frac{g_e}{6}$$
$$g_m = \frac{10}{6}$$
$$g_m = 1.67\ ms^{-2}$$

To Find

$$Length\ of\ pendulum\ on\ Earth = l_e = ?$$
$$Length\ of\ pendulum\ on\ Moon = l_m = ?$$

Solution

By using formula of time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l_e}{g_e}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l_e}{g_e}}$$

Taking square on both sides

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l_e}{g_e}}\right)^2$$
$$\frac{T^2}{4\pi^2} = \frac{l_e}{g_e}$$
$$\frac{T^2 g_e}{4\pi^2} = l_e$$
$$l_e = \frac{T^2 g_e}{4\pi^2}$$
$$l_e = \frac{(2)^2(10)}{4(3.14)^2}$$
$$l_e = \frac{40}{39.44}$$
$$\boldsymbol{l_e = 1.01\ m}$$

Now again by using formula of time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l_m}{g_m}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l_m}{g_m}}$$

Taking square on both sides

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l_m}{g_m}}\right)^2$$
$$\frac{T^2}{4\pi^2} = \frac{l_m}{g_m}$$
$$\frac{T^2 g_m}{4\pi^2} = l_m$$
$$l_m = \frac{T^2 g_m}{4\pi^2}$$
$$l_m = \frac{(2)^2(1.67)}{(4)(3.14)^2}$$
$$l_m = \frac{6.68}{39.44}$$
$$\boldsymbol{l_m = 0.17\ m}$$

10.2 A pendulum of length $0.99\ m$ is taken to the Moon by an astronaut. The period of the pendulum is $4.9\ s$. What is the value of g on the surface of the Moon? *(ALP)*

Given Data

$$Length\ of\ pendulum = l = 0.99\ m$$
$$Time\ period = T = 4.9\ s$$

To Find

$$Value\ of\ g\ on\ Moon = g_m = ?$$

Solution

By using formula of time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g_m}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{g_m}}$$

Taking square on both sides

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{g_m}}\right)^2$$
$$\frac{T^2}{4\pi^2} = \frac{l}{g_m}$$
$$T^2 g_m = 4\pi^2 l$$
$$g_m = \frac{4\pi^2 l}{T^2}$$
$$g_m = \frac{4(3.14)^2(0.99)}{(4.9)^2}$$
$$g_m = \frac{39.044}{24.01}$$
$$\boldsymbol{g_m = 1.63\ ms^{-2}}$$

10.3 Find the time periods of a simple pendulum of $1\ metre$ length, placed on Earth and on Moon. The value of g on the surface of Moon is $\left(\frac{1}{6}\right)^{th}$ of its value on Earth, where g is $10\ ms^{-2}$. *(ALP)*

Given Data

$$Length\ of\ pendulum = l = 1\ m$$
$$Value\ of\ g\ on\ Earth = g_e = 10\ ms^{-2}$$
$$Value\ of\ g\ on\ Moon = g_m = \frac{g_e}{6}$$

$$g_m = \frac{10}{6}$$
$$g_m = 1.67\ ms^{-2}$$

To Find

$$Time\ period\ on\ Earth = T_e = ?$$
$$Time\ period\ on\ Moon = T_m = ?$$

Solution

By using formula of time period

$$T_e = 2\pi\sqrt{\frac{l}{g_e}}$$
$$T_e = 2(3.14)\sqrt{\frac{1}{10}}$$
$$T_e = (6.28)(0.3162)$$
$$T_e = 1.99\ s$$
$$\boldsymbol{T_e \approx 2\ s}$$

Now again by using formula of time period

$$T_m = 2\pi\sqrt{\frac{l}{g_m}}$$
$$T_m = 2(3.14)\sqrt{\frac{1}{1.67}}$$
$$T_m = (6.28)(0.7738)$$
$$T_m = 4.85\ s$$
$$\boldsymbol{T_m \approx 4.9\ s}$$

10.4 A simple pendulum completes one vibration in two seconds. Calculate its length, when $g = 10\ ms^{-2}$. *(ALP)*

Given Data

$$Time\ period = T = 2\ s$$
$$Gravitational\ acceleration = g = 10\ ms^{-2}$$

To Find

$$Length\ of\ pendulum = l = ?$$

Solution

By using formula of time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

Taking square on both sides

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{g}}\right)^2$$
$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$
$$\frac{gT^2}{4\pi^2} = l$$
$$l = \frac{gT^2}{4\pi^2}$$
$$l = \frac{(10)(2)^2}{(4)(3.14)^2}$$
$$l = \frac{(10)(4)}{(4)(9.8596)}$$
$$l = \frac{40}{39.4384}$$
$$\boldsymbol{l = 1.02\ m}$$

10.5 If 100 *waves* pass through a point of a medium in 20 *seconds*, what is the frequency and the time period of the wave? If its wavelength is 6cm, calculate the wave speed.

Given Data

$$No.\ of\ waves = n = 100$$
$$Time\ aken = t = 20\ s$$
$$Wavelength = \lambda = 6\ cm$$
$$\lambda = 6 \times 10^{-2} m$$
$$\lambda = 0.06\ m$$

To Find

$$Frequency = f = ?$$
$$Time\ period = T = ?$$
$$Wave\ speed = v = ?$$

Solution

For frequency, we use

$$f = \frac{n}{t}$$
$$f = \frac{100}{20}$$
$$\boldsymbol{f = 5\ Hz}$$

For time period, we use

$$T = \frac{1}{f}$$
$$T = \frac{1}{5}$$
$$\boldsymbol{T = 0.2\ s}$$

For wave speed, we use

$$v = f\lambda$$
$$v = (5)(0.06)$$
$$\boldsymbol{v = 0.3\ ms^{-1}}$$

10.6 A wooden bar vibrating into the water surface in a ripple tank has a frequency of 12 *Hz*. The resulting wave has a wavelength of 3 *cm*. What is the speed of the wave?

Given Data

$$Frequency = f = 12\ Hz$$
$$Wavelength = \lambda = 3\ cm$$
$$\lambda = 3 \times 10^{-2} m$$
$$\lambda = 0.03\ m$$

To Find

$$Wave\ speed = v = ?$$

Solution

For wave speed, we use

$$v = f\lambda$$
$$v = (12)(0.03)$$
$$\boldsymbol{v = 0.36\ ms^{-1}}$$

10.7 A transverse wave produced on a spring has a frequency of 190 *Hz* and travels along the length of the spring of 90 *m*, in 0.5 *s*. (a) What is the period of the wave? (b) What is the speed of the wave? (c) What is the wavelength of the wave?

Given Data

$$Frequency = f = 190\ Hz$$

$$Length = d = 90\ m$$
$$Time = t = 0.5\ s$$

To Find

$$Time\ period = T = ?$$
$$Wave\ speed = v = ?$$
$$Wavelength = \lambda = ?$$

Solution

For time period, we use

$$T = \frac{1}{f}$$
$$T = \frac{1}{190}$$
$$T = 5.26 \times 10^{-3}$$
$$T = 0.00526$$
$$\boldsymbol{T \approx 0.01\ s}$$

For wave speed, we use

$$v = \frac{d}{t}$$
$$v = \frac{90}{0.5}$$
$$\boldsymbol{v = 180\ ms^{-1}}$$

For wavelength, we use

$$v = f\lambda$$
$$180 = (190)\lambda$$
$$\frac{180}{190} = \lambda$$
$$0.947 = \lambda$$
$$\boldsymbol{\lambda = 0.95\ m}$$

10.8 Water waves in a shallow dish are $6.0\ cm$ long. At one point, the water moves up and down at a rate of 4.8 oscillations per second. (a) What is the speed of the water waves? (b) What is the period of the water waves?

Given Data

$$Wavelength = \lambda = 6\ cm$$
$$\lambda = 6 \times 10^{-2} m$$
$$\lambda = 0.06\ m$$
$$Oscillations\ per\ second = f = 4.8\ Hz$$

To Find

$$Wave\ speed = v = ?$$
$$Time\ period = T = ?$$

Solution

For wave speed, we use

$$v = f\lambda$$
$$v = (4.8)(0.06)$$
$$\boldsymbol{v = 0.29\ ms^{-1}}$$

For time period, we use

$$T = \frac{1}{f}$$
$$T = \frac{1}{4.8}$$
$$\boldsymbol{T = 0.21\ s}$$

10. 9 At one end of a ripple tank 80 cm across, a $5\ Hz$ vibrator produces waves whose wavelength is $40\ mm$. Find the time the waves need to cross the tank.

Given Data

$$Length\ of\ tank = l = 80\ cm$$
$$d = 80 \times 10^{-2} m$$
$$d = 0.8\ m$$
$$Frequency = f = 5\ Hz$$
$$Wavelength = \lambda = 40\ mm$$
$$\lambda = 40 \times 10^{-3} m$$

To Find

$$Time\ taken = t = ?$$

Solution

First, we find the speed of wave

$$v = f\lambda$$
$$v = (5)(40 \times 10^{-3})$$
$$\boldsymbol{v = 0.2\ ms^{-1}}$$

Now, by using formula of speed

$$v = \frac{d}{t}$$
$$t = \frac{d}{v}$$
$$t = \frac{0.8}{0.2}$$
$$\boldsymbol{t = 4\ s}$$

10.10 What is the wavelength of the radio waves transmitted by an FM station at $90\ MHz$? Where $1M = 10^6$, and speed of radiowave is $3 \times 10^8\ ms^{-1}$.

Given Data

$$Frequency = f = 90\ MHz$$
$$f = 90 \times 10^6 H$$
$$Speed = v = 3 \times 10^8\ ms^{-1}$$

To Find

$$Wavelength = \lambda = ?$$

Solution

For wavelength, we use

$$v = f\lambda$$
$$3 \times 10^8 = (90 \times 10^6)\lambda$$
$$\frac{3 \times 10^8}{90 \times 10^6} = \lambda$$
$$3.33 = \lambda$$
$$\boldsymbol{\lambda = 3.33\ m}$$

Examples

10.1 Find the time period and frequency of a simple pendulum 1.0 m long at a location where $g = 10\ ms^{-2}$. *(ALP)*

Given Data

$$Length\ of\ pendulum = l = 1\ m$$
$$g = 10\ ms^{-2}$$

To Find

$$Time\ period = T = ?$$
$$Frequency = f = ?$$

Solution

By using formula of time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2(3.14)\sqrt{\frac{1}{10}}$$
$$T = (6.28)(0.3162)$$

$$T = 1.99\ s$$

For frequency, we use

$$T = \frac{1}{f}$$
$$f = \frac{1}{T}$$
$$f = \frac{1}{1.99}$$
$$\boldsymbol{f = 0.50\ Hz}$$

10.2 A wave moves on a slinky with frequency of $4\ Hz$ and wavelength of $0.4\ m$. What is the speed of the wave? *(ALP)*

Given Data

$$Frequency = f = 4\ Hz$$
$$Wavelength = \lambda = 0.4\ m$$

To Find

$$Wave\ speed = v = ?$$

Solution

For wave speed, we use

$$v = f\lambda$$
$$v = (4)(0.4)$$
$$\boldsymbol{v = 1.6\ ms^{-1}}$$

10.3 A student performs an experiment with waves in water. The student measures the wavelength of a wave to be 10 cm. By using a stopwatch and observing the oscillations of a floating ball, the student measures a frequency of 2 Hz. If the student starts a wave in one part of a tank of water, how long will it take the wave to reach the opposite side of the tank 2 m away?

Given Data

$$Frequency = f = 2\ Hz$$
$$Wavelength = \lambda = 10\ cm$$
$$\lambda = 10 \times 10^{-2} m$$
$$\lambda = 0.1\ m$$
$$Distance = d = 2\ m$$

To Find

$$Wave\ speed = v = ?$$
$$Time\ taken = t = ?$$

Solution

For wave speed, we use

$$v = f\lambda$$
$$v = (2)(0.1)$$
$$\boldsymbol{v = 0.2\ ms^{-1}}$$

Now, by using formula of speed

$$v = \frac{d}{t}$$
$$t = \frac{d}{v}$$
$$t = \frac{2}{0.2}$$
$$\boldsymbol{t = 10\ s}$$