Unit 4 Turning Effects of Force

Important Formulas

- **Resultant Force**

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

- **Angle**

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

- **x-component of force**

$$F_x = F\cos\theta$$

- **y-component of force**

$$F_y = F\sin\theta$$

- **Torque**

$$\tau = r \times F$$
$$\tau = rF\sin\theta$$

- **1st Condition of Equilibrium** $\sum F = 0$
- **2nd Condition of Equilibrium** $\sum \tau = 0$
- **Principle of moments**

$$clockwise\ moments\ = Anti\ clockwise\ moments$$

- **Weight** $w = mg$

4.1. A force of $200\ N$ is acting on a cart at an angle of 30° with the horizontal direction. Find the x and y-components of the force.

Given Data

$$Force = F = 200\ N$$
$$Angle = \theta = 30^\circ$$

To Find

$$x - component\ of\ force = F_x = ?$$
$$y - component\ of\ force = F_y = ?$$

Solution

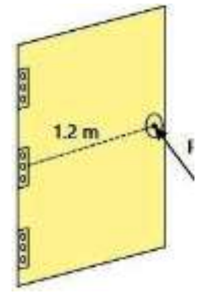
By using formula of F_x

$$F_x = F\cos\theta$$
$$F_x = 200\cos 30^\circ$$
$$F_x = (200)(0.866)$$
$$\boldsymbol{F_x = 173.2\ N}$$

Now by using formula of F_y

$$F_y = F\sin\theta$$
$$F_y = F\sin\theta$$
$$F_y = 200\sin 30^\circ$$
$$F_y = (200)(0.5)$$
$$\boldsymbol{F_y = 100\ N}$$

4.2. A force of $300\ N$ is applied perpendicularly at the knob of a door to open it as shown in the given figure. If the knob is $1.2\ m$ away from the hinge, what is the torque applied? Is it positive or negative torque?



Given Data

$$Force\ applied = F = 300\ N$$
$$Distance\ from\ hinge = r = 1.2\ m$$

Force is applied perpendicularly, so $\theta = 90^\circ$

To Find

$$Torque = \tau = ?$$

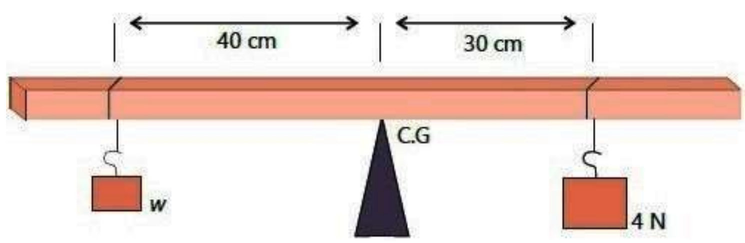
Solution

By using formula of torque, we have

$$\tau = rF\sin\theta$$
$$\tau = (1.2)(300)\sin 90^\circ$$
$$\tau = (1.2)(300)(1)$$
$$\boldsymbol{\tau = 360\ Nm}$$

Since the force turns the door counterclockwise, the torque is **positive**.

4.3. Two weights are hanging from a metre rule at the positions as shown in the given figure. If the rule is balanced at its centre of gravity (C.G), find the unknown weight w.



Given Data

$$Distance\ of\ unknown\ weight\ from\ C.G. = 40\ cm$$
$$= 40 \times 10^{-2}\ m$$
$$= 0.4\ m$$
$$Distance\ of\ 4\ N\ weight\ from\ C.G. = 30\ cm$$
$$= 30 \times 10^{-2}\ m$$
$$= 0.3\ m$$
$$Weight\ on\ right\ side = 4\ N$$

To Find

$$Unknown\ weight = w = ?$$

Solution

According to principle of moments,

$$clockwise\ moments\ = Anti\ clockwise\ moments$$
$$4 \times 0.3 = w \times 0.4$$
$$1.2 = 0.4w$$
$$\frac{1.2}{0.4} = w$$
$$3 = w$$

$$\boldsymbol{w = 3\ N}$$

4.4. A see-saw is balanced with two children sitting near either end. Child A weighs $30\ kg$ and sits $2\ metres$ away from the pivot, while child B weighs $40\ kg$ and sits $1.5\ metres$ from the pivot. Calculate the total moment on each side and determine if the sea-saw is in equilibrium.

Given Data

$$\begin{aligned}
&Mass\ of\ child\ A = m_A = 30\ N\\
&Distance\ of\ child\ A\ from\ pivot = r_A = 2\ m\\
&Mass\ of\ child\ B = m_B = 40\ kg\\
&Distance\ of\ child\ B\ from\ pivot = r_B = 1.5\ m\\
&Acceleration\ due\ to\ gravity = g = 10\ ms^{-2}
\end{aligned}$$

To Find

$$\begin{aligned}
&Total\ moment\ on\ each\ side = ?\\
&Determine\ if\ the\ see-saw\ is\ in\ equilibrium = ?
\end{aligned}$$

Solution

By using formula of the moment (torque)

Child A's Moment (Clockwise)

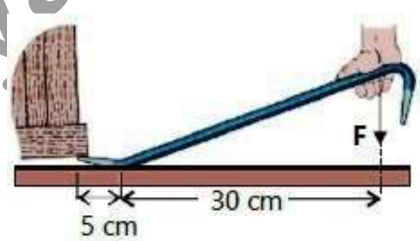
$$\begin{aligned}
\tau_A &= r_A \times F\\
\tau_A &= r_A \times w\\
\tau_A &= r_A mg \qquad \because w = mg\\
\tau_A &= (2)(30)(10)\\
\boldsymbol{\tau_A} &\boldsymbol{= 600\ Nm}
\end{aligned}$$

Child B's Moment (Anticlockwise)

$$\begin{aligned}
\tau_B &= r_B \times F\\
\tau_B &= r_B \times w\\
\tau_B &= r_B mg \qquad \because w = mg\\
\tau_B &= (1.5)(40)(10)\\
\boldsymbol{\tau_B} &\boldsymbol{= 600\ Nm}
\end{aligned}$$

Since both moments are **equal and opposite**, the **see-saw is in equilibrium**. $i.e\ \tau_A = \tau_B = 600\ Nm$

4.5. A crowbar is used to lift a box as shown in the given figure. If the downward force of 250 N is applied at the end of the bar, how much weight does the other end bear? The crowbar itself has negligible weight.



Given Data

$$\begin{aligned}
&Downward\ force = F_1 = 250\ N\\
&Distance\ from\ pivot\ to\ downward\ force = r_1\\
&\qquad = 30\ cm\\
&r_1 = 30 \times 10^{-2}\ m\\
&r_1 = 0.3\ m\\
&Distance\ of\ load\ from\ pivot\ = r_2 = 5\ cm\\
&r_2 = 5 \times 10^{-2}\ m\\
&r_2 = 0.05\ m
\end{aligned}$$

To Find

$$Weight\ bear\ at\ other\ end = F_2 = ?$$

Solution

According to principle of moments,

$$\begin{aligned}
clockwise\ moments\ &= Anti\ clockwise\ moments\\
r_1 \cdot F_1 &= r_2 \cdot F_2\\
(0.3)(250) &= (0.05)(F_2)\\
\frac{(0.3)(250)}{0.05} &= F_2\\
15000 &= F_2\\
\boldsymbol{F_2} &\boldsymbol{= 15000\ N}
\end{aligned}$$

4.6. A 30 cm long spanner is used to open the nut of a car. If the torque required for it is 150 N m, how much force F should be applied on the spanner as shown in the given figure.

Given Data

$$\begin{aligned}
&Length\ of\ spanner = r = 30\ cm\\
&r = 30 \times 10^{-2}\ m\\
&r = 0.3\ m\\
&Torque = \tau = \ 150\ Nm
\end{aligned}$$

To Find

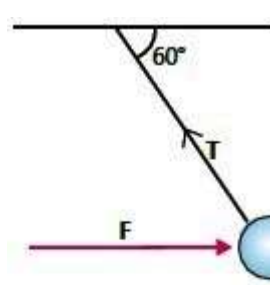
$$\text{Force} = F = ?$$

Solution

By using formula of torque, we have

$$\begin{aligned}
\tau &= rF\\
(150) &= (0.3)(F)\\
\frac{150}{0.3} &= F\\
500 &= F\\
\boldsymbol{F} &\boldsymbol{= 500\ N}
\end{aligned}$$

4.7. A 5 N ball hanging from a rope is pulled to the right by a horizontal $force\ F$. The rope makes an angle of 60° with the ceiling, as shown in the given figure. Determine the magnitude of $force\ F$ and $tension\ T$ in the string.



Given Data

$$\begin{aligned}
&Weight\ of\ ball = w = 5\ N\\
&Angle\ with\ ceiling = \theta = 60^\circ
\end{aligned}$$

To Find

$$\begin{aligned}
&Horizontal\ force = F = ?\\
&Tension\ in\ string = T = ?
\end{aligned}$$

Solution

The tension T in the rope has two components $i.e$

$$T_x = T\cos\theta$$
$$T_y = T\sin\theta$$

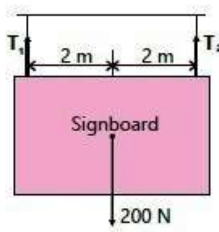
The upward force T_y equals the ball's weight w because it is in balance.

$$T_y = w$$
$$T\sin\theta = w$$
$$T\sin 60^\circ = 5$$
$$T = \frac{5}{\sin 60^\circ}$$
$$T = \frac{5}{0.866}$$
$$\boldsymbol{T = 5.8\ N}$$

The horizontal force F is equal to the tension's horizontal part T_x

$$F = T_x$$
$$F = T\cos\theta$$
$$F = (5.8)\cos 60^\circ$$
$$F = (5.8)(0.5)$$
$$\boldsymbol{F = 2.9\ N}$$

4.8. A signboard is suspended by means of two steel wires as shown in the given figure. If the weight of the board is $200\ N$, what is the tension in the strings?



Given Data

$$Weight\ of\ the\ board = w = 200\ N$$

To Find

$$Tension\ in\ first\ string = T_1 = ?$$
$$Tension\ in\ second\ string = T_2 = ?$$

Solution

Since wires are symmetrically placed so tension in each string will be equal $i.e\ T = T_1 = T_2$

As signboard is in equilibrium. So

$$T = w$$
$$T_1 + T_2 = w$$
$$T + T = 200$$
$$2T = 200$$
$$T = \frac{200}{2}$$
$$T = 100\ N$$
$$\boldsymbol{T_1 = 100\ N}$$
$$\boldsymbol{T_2 = 100\ N}$$

4.9. One girl of $30\ kg$ mass sits $1.6\ m$ from the axis of a see-saw. Another girl of mass $40\ kg$ wants to sit on the other side, so that the see-saw may remain in equilibrium. How $far\ away\ from$ the axis, the other girl may sit?

Given Data

$$Mass\ of\ first\ girl = m_1 = 30\ kg$$
$$Distance\ of\ first\ girl\ from\ axis = r_1 = 1.6\ m$$
$$Mass\ of\ second\ girl = m_2 = 40\ kg$$

To Find

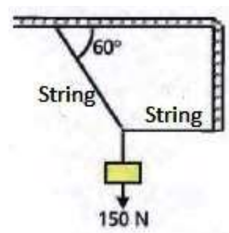
$$Distance\ of\ second\ girl\ from\ axis = r_2 = ?$$

Solution

According to principle of moments,

$$clockwise\ moments = Anti\ clockwise\ moments$$
$$r_1 \cdot F_1 = r_2 \cdot F_2$$
$$r_1 \cdot m_1 g = r_2 \cdot m_2 g$$
$$(1.6)(30)(10) = (r_2)(40)(10)$$
$$\frac{(1.6)(30)(10)}{(40)(10)} = r_2$$
$$1.2 = r_2$$
$$\boldsymbol{r_2 = 1.2\ m}$$

4.10. Find the tension in each string of the as shown in the given figure, if the block weighs 150 N.



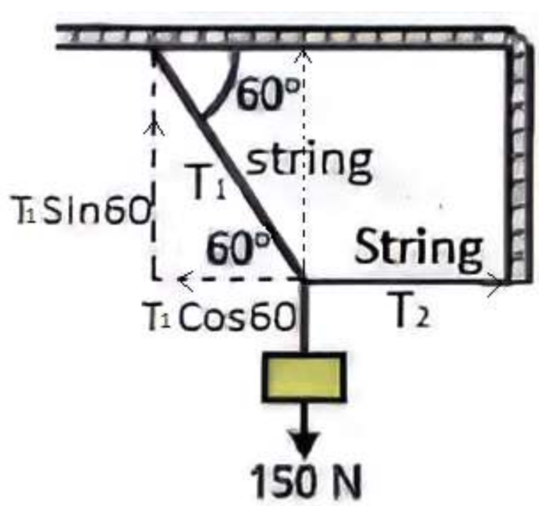
Given Data

$$Weight\ of\ block = w = 150\ N$$

To Find

$$Tension\ in\ first\ string = T_1 = ?$$
$$Tension\ in\ second\ string = T_2 = ?$$

Solution



As block is in equilibrium. So, the vertical component of T_1 balances the weight w

$$T_1\sin\theta = w$$
$$T_1\sin 60^\circ = 150$$
$$T_1 = \frac{150}{\sin 60^\circ}$$
$$T_1 = \frac{150}{0.866}$$

$$\boldsymbol{T_1 = 173.21\ N}$$

Now, the horizontal component of T_1 is balanced by T_2

$$T_1 \cos\theta = T_2$$
$$T_1 \cos 60^\circ = T_2$$
$$(173.21)\cos 60^\circ = T_2$$
$$(173.21)(0.5) = T_2$$
$$86.6 = T_2$$
$$\boldsymbol{T_2 = 86.6\ N}$$